## Chapter 5

Properties of Triangles

Section 1
Perpendiculars and Bisectors

## GOAL 1: Using Properties of Perpendicular Bisectors

In Lesson 1.5, we learned that a segment bisector intersects a segment at is midpoint. A segment, ray, line, or plane that is perpendicular to a segment at its midpoint is called a $\qquad$ perpendicular bisector $\qquad$ .

$\overleftrightarrow{C P}$ is a $\perp$ bisector of $\overline{A B}$.

The construction below shows how to draw a line that is perpendicular to a given line or segment at a point $P$. You can use this method to construct a perpendicular bisector of a segment as described below the activity.

Use these steps to construct a line that is perpendicular to a given line $m$ and that passes through a given point $P$ on $m$.

(1) Place the compass point at $P$. Draw an arc that intersects line $m$ twice. Label the intersections as $A$ and $B$.

(2) Use a compass setting greater than $A P$. Draw an arc from $A$. With the same setting, draw an arc from $B$. Label the intersection of the arcs as $C$.

(3) Use a straightedge to draw $\overleftrightarrow{C P}$. This line is perpendicular to line $m$ and passes through $P$.

You can measure <CPA on your construction to verify that the constructed line is perpendicular to the given line $m$. In the construction, $\overleftrightarrow{C P} \perp \overrightarrow{A B}$ and $P A=P B$, so $\stackrel{C P}{ }$ is the perpendicular bisector of $\overrightarrow{A B}$.

A point is $\qquad$ equidistant from two points $\qquad$ if its distance from each point is the same. In the construction on the previous slide, C is equidistant from $A$ and $B$ because $C$ was drawn so that $C A=C B$.

Theorem 5.1 states that any point on the perpendicular bisector $\overleftrightarrow{C P}$ in the construction is equidistant from $A$ and $B$, the endpoints of the segment. The converse helps you prove that a given point lies on a perpendicular bisector.

## THEOREMS

## theorem 5.1 Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

If $\overleftrightarrow{C P}$ is the perpendicular bisector of $\overline{A B}$, then $C A=C B$.


$$
C A=C B
$$

## theorem 5.2 Converse of the Perpendicular Bisector Theorem

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

If $D A=D B$, then $D$ lies on the perpendicular bisector of $\overline{A B}$.

$D$ is on $\overleftrightarrow{C P}$

Plan for Proof of Theorem 5.1 Refer to the diagram for Theorem 5.1 above. Suppose that you are given that $\overleftrightarrow{C P}$ is the perpendicular bisector of $\overline{A B}$. Show that right triangles $\triangle A P C$ and $\triangle B P C$ are congruent using the SAS Congruence Postulate. Then show that $\overline{C A} \cong \overline{C B}$.

Exercise 28 asks you to write a two-column proof of Theorem 5.1 using this plan for proof. Exercise 29 asks you to write a proof of Theorem 5.2.

## Example 1: Using Perpendicular Bisectors

In the diagram shown, $\overleftrightarrow{M N}$ is the perpendicular bisector of $\overrightarrow{\mathrm{ST}}$.
a) What segment lengths in the diagram are equal?

b) Explain why Q is on $\overleftrightarrow{\mathrm{MN}}$.

Q is equidistant from $\mathrm{T} \& \mathrm{~S}(\mathrm{TQ}=\mathrm{SQ}) \rightarrow 5.2 \rightarrow \mathrm{Q}$ is on MN

GOAL 2: Using Properties of Angle Bisectors

The $\qquad$ distance from a point to a line $\qquad$ is defined as the length of the perpendicular segment from the point to the line. For instance, in the diagram shown, the distance between the point $Q$ and the line $m$ is $Q P$.


When a point is the same distance from one line as it is from another line, then the point is $\qquad$ equidistant from the two lines $\qquad$ (or rays or segments). The theorems below show that a point in the interior of an angle is equidistant from the sides of the angle if and only if the point is on the bisector of the angle.

## THEOREMS

## THEOREM 5.3 Angle Bisector Theorem

If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.

$$
\text { If } m \angle B A D=m \angle C A D, \text { then } D B=D C \text {. }
$$



$$
D B=D C
$$

theorem 5.4 Converse of the Angle Bisector Theorem
If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.

$$
\text { If } D B=D C \text {, then } m \angle B A D=m \angle C A D \text {. }
$$



$$
m \angle B A D=m \angle C A D
$$

A paragraph proof of Theorem 5.3 is given in Example 2. Exercise 32 asks you to write a proof of Theorem 5.4.

## Example 2: Proof of Theorem 5.3

Given: $D$ is on the bisector of $\angle B A C . \overline{D B} \perp \overrightarrow{A B}, D C \perp \overrightarrow{A C}$ Prove: DB = DC


Plan for Proof: Prove that $\triangle A D B \cong \triangle A D C$. Then conclude that $\mathrm{DB} \cong \overline{\mathrm{DC}}$, so $\mathrm{DB}=\mathrm{DC}$. (6 sentences total)
$D$ is on the bisector of $\angle B A C$. $D B \perp A B, D C \perp A C$ <BAD cong. <CAD
$<B$ and $<C$ are right angles
<B cong. <C
DA cong. DA
Tri. ADB cong. Tri. ADC
DB cong. DC
$D B=D C$
given
def. of < bisector def. of perp. lines right < congruence thm.
reflexive/o.s.
AAS
CPCTC
def. of cong. segments

## Example 3: Using Angle Bisectors

Some roofs are built with wooden trusses that are assembled in a factory and shipped to the building site. In the diagram of the roof truss, you are given that $\overrightarrow{A B}$ bisects <CAD and that <ACB and <ADB are right angles. What can you say about $B C$ and $B D$ ?

$<\mathrm{CAB} \&<\mathrm{DAB}$ are congruent $\rightarrow<\mathrm{C} \&<\mathrm{D}$ are right angles $\rightarrow \mathrm{BC} \& \mathrm{BD}$ show the distance from $B$ to $A C$ and $B$ to $A D \rightarrow B$ is on the bisector $\rightarrow$ it is equidistant

EXIT SLIP

